$$=\sqrt{\{[a^2+2\sqrt{(abc)}][b^2+2\sqrt{(abc)}][c^2+2\sqrt{(abc)}]\}}.$$

This gives us the values of the sides.

Otherwise draw $AO$ and produce $AO$ to $O_1$ so that $OO_1=2\sqrt{(bc/a)}$. Upon $OO_1$ as diameter describe a circle. With $O$ as a center and $b$ as a radius describe an arc cutting the circle in $B$. Similarly, with $O$ as center and $c$ as radius, draw an arc cutting the circle in $C$. Join $BC$, $AC$, $AB$, then $ABC$ is the triangle required. For $O_1$ is the ex-center opposite $A$ by construction as follows:

$AO_1=p/\cos\frac{1}{2}A$. $\therefore$ $AOAO_1=yz=AO^2+2\sqrt{(AO.BO.CO)}$.

$\therefore$ $AO_1=AO+2\sqrt{[(BO.CO)/AO]}$.

## CALCULUS.

121. Proposed by W. W. LANDIS. A. M., Professor of Mathematics and Astronomy, Dickinson College, Carlisle, Pa.

Solve the differential equation $\left[\dfrac{d}{dx}+b\right]^n y=\cos ax$.

Solution by LON C. WALKER, A. M., Petaluma High School, Petaluma, Cal., and LEWIS NEIKIRK, B. S., Boulder, Col.

$$\left[\frac{d}{dx}+b\right]^n y=\cos ax.$$

$\left[\dfrac{d}{dx}+b\right]^n$ has $n$ roots each $=-b$.

$\therefore$ Comp. Factor $=e^{-bx}(c_1+c_2x+c_3x^2+c_4x^3+\dots c_nx^{n-1})$.

$$\frac{1}{\left[\dfrac{d}{dx}+b\right]^n}\cos ax=\frac{\left[\dfrac{d}{dx}-b\right]^n}{\left[\dfrac{d}{dx}-b^2\right]^n}\cos ax=\left\{\frac{b-\dfrac{d}{dx}}{a^2+b^2}\right\}^n\cos ax.$$

Let $n=1$. $\therefore$ $\dfrac{b-d/dx}{a^2+b^2}\cos ax=\dfrac{1}{a^2+b^2}(b\cos ax+a\sin ax)$

$$=\frac{1}{(a^2+b^2)^{\frac{1}{2}}}\left[\frac{b\cos ax+a\sin ax}{\sqrt{(a^2+b^2)}}\right]\dots.(1).$$

Put $\theta=\cot^{-1}b/a$, then $\sin\theta=\dfrac{a}{\sqrt{(a^2+b^2)}}$, and $\cos\theta=\dfrac{b}{\sqrt{(a^2+b^2)}}$.

$\therefore$ (1) reduces to

$$\frac{b-d/dx}{a^2+b^2}\cos ax=(a^2+b^2)^{-\frac{1}{2}}(\cos\theta\cos ax+\sin\theta\sin ax)=(a^2+b^2)^{-\frac{1}{2}}\cos(ax-1\theta).$$

When $n=2$, $\frac{b-d/dx}{a^2+b^2}\left[\frac{1}{(a^2+b^2)^{\frac{1}{2}}}\cos(ax-\theta)\right]$

$$=\frac{1}{a^2+b^2}\left[\frac{b\cos(ax-\theta)+a}{\sqrt{(a^2+b^2)}}\sin(ax+\theta)\right]$$

$$=\frac{1}{a^2+b^2}\left[\cos\theta\cos(ax-\theta)+\sin\theta\sin(ax-\theta)\right]=(a^2+b^2)^{-\frac{2}{2}}\cos(ax-2\theta).$$

When $n=3$, $\therefore$ $\frac{b-d/dx}{a^2+b^2}\left[\frac{1}{(a^2+b^2)^{\frac{1}{2}}}\cos(ax-\theta)\right]$

$$=\frac{1}{a^2+b^2}\left[\frac{b\cos(ax-2\theta)a\sin(ax-2\theta)}{\sqrt{(a^2+b^2)}}\right]=(a^2+b^2)^{-\frac{3}{2}}\cos(ax-3\theta).$$

This method holds for $(n-1)$ terms.

Hence $y=e^{-bx}(c_1+c_2x+c_3x^2+c_4x^3+\ldots.c_nx^{n-1})$

$$+(a^2+b^2)^{-\frac{1}{2}n}\cos[ax-n\cot^{-1}(b/a)].$$

SECOND SOLUTION.

Put $D=\frac{d}{dx}$, then we have $\frac{1}{(D+b)^n}\cos ax=\frac{(D-b)^n}{(D^2-b^2)^n}\cos ax$

$$=\frac{(-1)^n(b-D)^n}{(a^2+b^2)^n}\cos ax.$$

The numerator$=(-1)^n\left[b^n\cos ax+nab^{-1}\sin ax)-\frac{n(n-1)}{2!}a^2b^{n-2}\cos ax\ldots\right]$

$$=(-1)^n\cos[ax-n\cot^{-1}(b/a)]$$

The value of $y$ is the same as in I.

Also solved by *WILLIAM HOOVER*, *G. B. M. ZERR*, and the *PROPOSER*.

Professor Landis remarks that the exponent of $a^2+b^2$ is given $\frac{1}{2}n$ in Johnson's *Differential Equations* (problem 17, page 122) instead of the correct result $-\frac{1}{2}n$.

**122.** Proposed by **B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.**

Solve the differential equation $(y-x)\sqrt{(1+x^2)}\frac{dy}{dx}=n(1+y^2)^{\frac{3}{2}}$

**I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.**

Let $x=\tan\theta$, $y=\tan\varphi$, then $\sin(\phi-\theta)d\varphi=nd\theta\ldots.(1)$.